1-1-2017

Testing for differences in technical efficiency among groups within an industry

Phuong Le
University of Wollongong, tpl375@uowmail.edu.au

Charles Harvie
University of Wollongong, charvie@uow.edu.au

Amir Arjomandi
University of Wollongong, amira@uow.edu.au

This study generalizes the test performed by Simar and Zelenyuk (2007) to examine differences in the technical efficiency among (Formula presented.) groups within an industry (where (Formula presented.)). For this purpose, the (Formula presented.) groups are divided into pairs and each group is compared with all other (Formula presented.) groups. The (Formula presented.) groups can then be classified into three cohorts: those performing better, equally and worse, relative to the benchmark group. For illustration purposes, annual data for Vietnamese banks covering the period 2005--2012 is used.

Testing for differences in technical efficiency among groups within an industry

Phuong Thanh Le, Charles Harvie, Amir Arjomandi

School of Accounting, Economics and Finance, University of Wollongong, Wollongong, NSW 2522, Australia

This study generalises the test performed by Simar and Zelenyuk (2007) to examine differences in the technical efficiency among L groups within an industry (where $L \geq 2$). For this purpose the L groups are divided into pairs and each group is compared with all other $L - 1$ groups. The $L - 1$ groups can then be classified into three cohorts: those performing better; equally; or worse, relative to the benchmark group. For illustration purposes, annual data for Vietnamese banks covering the period 2005–2012 is used.

***Keywords*:** data envelopment analysis (DEA); bootstrap; technical efficiency; group efficiency

***JEL Classification*:** G21; D24

I. Introduction

The difference in DEA efficiency among groups can be identified using non-parametric or parametric tests. Non-parametric tests such as the Mann−Whitney test (for two groups) and the Kruskal–Wallis test (for more than two groups) are relatively easier to use because, unlike parametric tests, they do not impose any assumptions on the distribution of the inefficiency deviation (Sueyoshi and Aoki, 2001). Nonetheless, such tests are based on DEA estimates rather than true efficiencies and so issues of finite-sample bias and dependency are ignored (Simar and Zelenyuk, 2007). With the parametric approaches the inefficiency deviations are assumed to follow a half-normal or exponential distribution (Banker, 1993); thus, the appropriateness of these assumptions may need to be considered.

To avoid the above-mentioned issues, bootstrap techniques can be used to identify the sampling distribution of DEA efficiency scores (Sueyoshi and Aoki, 2001). Simar and Zelenyuk (2007) use a subsampling bootstrap technique to identify an efficiency sampling distribution when comparing two groups' DEA efficiency scores, overcoming the nature of bias and dependency of the estimates. Although their approach has been valuable in identifying better performers, there is a need to conduct the test for more than two groups in areas such as different ownership types (foreign, private, public, etc.) and different industries (textile, electronics, automotive, etc.). Such analyses can provide further help to policy makers to assess the effectiveness of strategies for economic growth and prioritising resources. Therefore, the objective of this study is to generalise the application of the Simar and Zelenyuk (2007) test to more than two groups.

II. Methodology

We consider an industry consisting of *n* firms belonging to L groups divided into $(L-1)L/2$ pairs. The benchmarking group will be compared with the $L-1$ remaining groups. For any comparison there are two groups (i and j). The hypothesis to compare any two groups is:

$$H_o: \bar{\delta}^i = \bar{\delta}^j \text{ against } H_1: \bar{\delta}^i \neq \bar{\delta}^j \; (i \neq j \text{ and } i, j: 1, \dots, L)$$

where $\bar{\delta}^i$ and $\bar{\delta}^j$ are the aggregate efficiency scores of groups i and j respectively.[1] The aggregate efficiency of a group is equal to the weighted sum of firm efficiency scores with the weights calculated on the basis of revenue shares for the entire group.

Due to the multiplicative nature of efficiency, Simar and Zelenyuk (2007) propose estimating the ratio of group i and group j's aggregate efficiency scores: $RD_{i,j} = \bar{\delta}^i / \bar{\delta}^j$ and its DEA estimate is computed as $\widehat{RD}_{i,j} = \bar{\hat{\delta}}^i / \bar{\hat{\delta}}^j$. For elaborating a sampling distribution of $RD_{i,j}$, the bootstrap $RD_{i,j}$ is computed as below:

$$\widehat{RD}^*_{i,j,b} = \frac{\overline{\hat{\delta}^*_b}^i}{\overline{\hat{\delta}^*_b}^j}, b = 1, \dots, B$$

[1] Our DEA model is specified by the conditions of output orientation and variable returns to scale.

where $\overline{\hat{\delta}_b^*}^i$ and $\overline{\hat{\delta}_b^*}^j$ are the bootstrap estimates of the aggregate efficiency scores of groups i and j ($i \neq j$ and $i, j: 1, \dots, L$) at the b bootstrap iteration of the total B times.

Then, the bias-corrected estimates of $RD_{i,j}$ and their bootstrap estimates can be computed as:

$$\widetilde{RD}_{i,j} = 2\widehat{RD}_{i,j} - \frac{1}{B}\sum_{b=1}^{B} \widehat{RD}_{i,j,b}^*$$

and

$$\widetilde{RD}_{i,j,b} = 2\widehat{RD}_{i,j} - \widehat{RD}_{i,j,b}^*$$

Based on these sorted values of $\widetilde{RD}_{i,j,b}$ the lower and upper bounds (the confidence interval) of $RD_{i,j}$ at α significance degree can be identified. We can then conclude which hypothesis is to be rejected using the following rule:

Reject H_o if the confidence interval for $RD_{i,j}$ does not overlap with unity, and do not reject otherwise. In particular, if the confidence interval lies above or below unity then we can conclude that $\bar{\delta}^i > \bar{\delta}^j$ or $\bar{\delta}^i < \bar{\delta}^j$.

The values of $\overline{\hat{\delta}_b^*}^i$, $\overline{\hat{\delta}_b^*}^j$ and $\widehat{RD}_{i,j,b}^*$ are computed under a subsampling bootstrap process as below: [2,3]

Step 1:

(i) Using the original sample $\Xi_n = \{(x^k, y^k): k = 1, \dots, n\}$ and DEA in order to obtain estimates of the true efficiency scores $\{\delta(x^k, y^k): k = 1, \dots, n\}$, denote these estimates as $\{\hat{\delta}^k: k = 1, \dots, n\}$.

[2] For details on subsampling, see Simar and Zelenyuk (2007).

[3] The algorithm of the subsampling bootstrap process for mean DEA efficiency scores is the same with equal weights.

(ii) Partition the original sample into L distinct groups $\Xi_{n_l} = \{(x^{l,k}, y^{l,k}) : k = 1, \dots, n_l\}$ and $\{\hat{\delta}^{l,k} : k = 1, \dots, n_l\}, l \in \{1; \dots, L\}$ representing the corresponding groups within the sample. Obtain estimates of the aggregate efficiency scores, $\bar{\hat{\delta}}^l$, for each group $l \in \{1, \dots, L\}$.

$$\bar{\hat{\delta}}^l = \sum_{k=1}^{n_l} \hat{\delta}^{l,k} . S^{l,k} \tag{1}$$

where $S^{l,k} = \frac{p y^{l,k}}{p \sum_{k=1}^{n_l} y^{l,k}}, k = 1, \dots, n_l$; and p is the price vector; n_l is the number of observations of the l group; and $y^{l,k}$ is the output vector of the k firm belonging to the l group.

If price information is unavailable, then we can use:

$$S^{l,k} = \frac{\frac{1}{M} \sum_{m=1}^{M} \frac{y_m^{l,k}}{\sum_{l=1}^{L} \sum_{k=1}^{n_l} y_m^{l,k}}}{S^l} \tag{2}$$

where $S^l = \frac{1}{M} \sum_{m=1}^{M} \frac{\sum_{k=1}^{n_l} y_m^{l,k}}{\sum_{l=1}^{L} \sum_{k=1}^{n_l} y_m^{l,k}}$ and M is the number of directions of vector y.

Accordingly, we can identify the $(L-1)L/2$ DEA-based ratios of the group i aggregate efficiency score over that of group j ($\widehat{RD}_{i,j} = \bar{\hat{\delta}}^i / \bar{\hat{\delta}}^j$).

Step 2:

For a bootstrap procedure, conduct subsampling with replacement independently on each group l to create L bootstrap sequences, $\Xi_{s_l,b}^* = \{(x_b^{*l,k}, y_b^{*l,k}) : k = 1, \dots, s_l\}$, where s_l is the size of the bootstrap subsample and $s_l \equiv (n_l)^\kappa$, $\kappa < 1$, $l \in \{1, \dots, L\}$. By combining these L sequences a bootstrap sample of the whole industry can be obtained, and then by applying DEA and Equation (1) or (2) the bootstrap aggregate efficiency scores of each group ($\overline{\hat{\delta}_b^*}^l$) for $(l: 1, \dots, L)$ can be calculated. Accordingly, we calculate the $(L-1)L/2$ ratios for the group i bootstrap aggregate efficiency score over that of group j as $\widehat{RD}^*_{i,j,b} = \overline{\hat{\delta}^*_b}^i / \overline{\hat{\delta}^*_b}^j$, for $(i \neq j;\ i, j: 1, \ldots, L)$.

Step 3:

The bootstrap procedure is repeated B times to obtain B values of the bootstrap aggregate efficiency scores for each group ($\overline{\hat{\delta}^*_b}^l$, for $l: 1, \ldots, L$) and B values of the group i bootstrap aggregate efficiency score over that of group j ($\widehat{RD}^*_{i,j,b}$).[4]

III. An illustration

We now implement an empirical analysis using the generalised test based upon data for Vietnamese banks over the period 2005–2012. Beresford (2008) believes that the business environment in Vietnam is heterogeneous due to the state's favourable treatment of state-owned enterprises whilst discriminating against the private sector. In the context of the banking sector this issue can be examined by analysing the impact of ownership on bank efficiency.

The sample includes 232 bank-year observations and can be divided into four groups: state-owned commercial banks (SOCB); foreign and joint venture banks (FJVB); urban joint stock banks (JSB); and banks transformed from rural to urban joint stock banks (TJSB). These banks account for 90 per cent of the sector's assets. To estimate the DEA model two inputs (interest and non-interest expenses) and two outputs (interest and non-interest incomes) are utilised. The data are collected from the financial statements of each bank.

Table 1 presents the results of the aggregate and mean efficiencies of the various bank groups. The columns "DEA Estimation" and "Bias Correction Estimation" present estimates

[4] The MATLAB codes used by Simar and Zelenyuk (2007) for an industry with two groups were modified for an L-group industry.

obtained by basic DEA and subsampling-bootstrap DEA, respectively. At each level of significance (1%, 5% and 10%), there are two columns presenting the lower and upper bounds of relevant estimators. For example, the value of the JSB aggregate efficiency score (Agg.Eff. JSB) is 1.2032 when estimated by basic DEA and 1.2907 using bootstrap DEA. Its lower and upper bounds are 1.2211 and 1.3482, respectively, at the 10% level of significance.

Table 2 uses the aggregate and mean efficiency ratios in Table 1 to classify the compared groups into different cohorts. In this case the JSB group is the benchmarking group under the aggregate criterion. First of all JSBs are compared with TJSBs. At the 1% level of significance the confidence interval of RD_ag JSB/TJSB (ratio of the JSB aggregate efficiency score over that of TJSBs) is between 0.7710 and 0.9729. Both the lower and upper bounds are smaller than unity suggesting that the ratio is significantly smaller than unity. We can therefore conclude that the aggregate efficiency score of JSBs is significantly smaller than that of TJSBs and JSBs perform more efficiently than TJSBs. Secondly, in the JSB/SOCB comparison, the ratio of the JSB aggregate efficiency score over that of SOCBs (RD_ag JSB/SOCB) is between 1.1026 and 1.2858 at the 1% level of significance. Both the lower and upper bounds are greater than unity suggesting that the JSB aggregate efficiency score is larger than that of SOCBs and that the former is less efficient than the latter. Lastly, at the 5% significance level the lower and upper bounds of the ratio of the JSB aggregate efficiency score over that of FJVBs are 1.0082 and 1.2458 indicating a superior performance by foreign and joint venture banks.[5]

[Tables 1 and 2 here]

[5] At any significance level, if the lower bound is smaller than unity, while the upper bound is greater, the efficiency of the groups being compared will be equal.

IV. Conclusion

This paper suggests a way to compare efficiency in industries with an arbitrary number of groups by generalising the bootstrap-based test of Simar and Zelenyuk (2007) which can be used in any study that requires efficiency comparisons between more than two groups. In fact, a number of further applications can be conducted in many areas of economics other than the banking industry (e.g. small and medium firm performance across countries or farm efficiency in different climatic regions).

Table 1: Aggregate and mean efficiencies of the groups and their ratios

	DEA Estimation	Bias Correction Estimation	Confidence Intervals					
			90%		95%		99%	
Agg.Eff. JSB	1.2032	1.2907	1.2211	1.3482	1.2074	1.3548	1.1814	1.3638
Agg.Eff. TJSB	1.3432	1.4680	1.3718	1.5551	1.3555	1.5712	1.3175	1.5939
Agg.Eff. SOCB	1.0443	1.0677	1.0478	1.0819	1.0419	1.0833	1.0290	1.0853
Agg.Eff. FJVB	1.1174	1.1538	1.0378	1.2186	1.0148	1.2235	0.9750	1.2317
M.Eff. JSB	1.2860	1.3995	1.3179	1.4732	1.3050	1.4826	1.2700	1.4968
M.Eff. TJSB	1.4055	1.5623	1.4189	1.6720	1.3845	1.6891	1.3110	1.7123
M.Eff. SOCB	1.1255	1.1712	1.1067	1.2253	1.0926	1.2332	1.0699	1.2428
M.Eff. FJVB	1.2988	1.4350	1.3078	1.5481	1.2808	1.5653	1.2241	1.5830
RD_ag JSB/TJSB	0.8958	0.8747	0.8121	0.9357	0.8004	0.9465	0.7710	0.9729
RD_ag JSB/SOCB	1.1522	1.2114	1.1433	1.2655	1.1303	1.2719	1.1026	1.2858
RD_ag JSB/FJVB	1.0768	1.1190	1.0234	1.2292	1.0082	1.2458	0.9813	1.2860
RD_ag TJSB/SOCB	1.2863	1.3790	1.2864	1.4604	1.2723	1.4767	1.2267	1.5028
RD_ag TJSB/FJVB	1.2021	1.2746	1.1606	1.3973	1.1381	1.4190	1.1033	1.4592
RD_ag SOCB/FJVB	0.9346	0.9224	0.8631	1.0152	0.8572	1.0313	0.8457	1.0584
RD_mean JSB/TJSB	0.9150	0.8886	0.8163	0.9762	0.8033	0.9967	0.7661	1.0346
RD_mean JSB/SOCB	1.1426	1.1984	1.1087	1.2809	1.0921	1.2939	1.0463	1.3123
RD_mean JSB/FJVB	0.9902	0.9684	0.8585	1.0781	0.8407	1.0953	0.7998	1.1309
RD_mean TJSB/SOCB	1.2487	1.3401	1.2034	1.4505	1.1599	1.4676	1.1021	1.4960
RD_mean TJSB/FJVB	1.0821	1.0870	0.9448	1.2138	0.9144	1.2371	0.8234	1.2718
RD_mean SOCB/FJVB	0.8666	0.8009	0.6932	0.9054	0.6766	0.9222	0.6502	0.9680

Notes: Agg.Eff. and M.Eff are aggregate and mean efficiencies, respectively.
RD_ag JSB/SOCB and RD_mean JSB/SOCB are ratios of the aggregate and mean technical efficiency scores of JSBs over those of SOCBs, respectively. Same logic is used for the other pairs of bank groups.

Table 2: Comparing the different bank groups' technical efficiencies

	Aggregate Criterion				Mean Criterion			
	JSB	TJSB	SOCB	FJVB	JSB	TJSB	SOCB	FJVB
Worse	TJSB***	−	TJSB*** JSB***	TJSB*** JSB**	TJSB**	−	JSB*** TJSB*** FJVB***	−
Equal	−	−	FJVB	SOCB	FJVB	FJVB	−	JSB TJSB
Better	SOCB*** FJVB**	JSB*** SOCB*** FJVB***	−	−	SOCB***	JSB** SOCB***	−	SOCB***

Notes: ** $p < 0.05$, *** $p < 0.01$

References

Banker, R. D. (1993) Maximum likelihood, consistency and data envelopment analysis: a statistical foundation, *Management Science*, **39**, 1265−1273. doi: http://dx.doi.org/10.1287/mnsc.39.10.1265

Beresford, M. (2008) Doi Moi in review: the challenges of building market socialism in Vietnam, *Journal of Contemporary Asia*, **38**, 221−243. doi: 10.1080/00472330701822314

Simar, L. and Zelenyuk, V. (2007) Statistical inference for aggregates of Farrell-type efficiencies, *Journal of Applied Econometrics*, **22**, 1367−1394. doi: 10.1002/jae.991

Sueyoshi, T. and Aoki, S. (2001) A use of a nonparametric statistic for DEA frontier shift: the Kruskal and Wallis rank test, *Omega*, **29**, 1−18. doi: 10.1016/S0305-0483(00)00024-4